Solve the following triangles. **Sketch and label triangles with your final answers.**   SCORE: _____ / 12 PTS
If no such triangle exists, write "DNE". If more than one triangle is possible, solve for all possible triangles.

[a] $\triangle TSA$
  if $t = 4.3$, $s = 7.2$ and $a = 3.1$

[b] $\triangle FDR$
  if $r = 6.2$, $d = 4.7$ and $D = 41°$

[c] $\triangle PBS$
  if $p = 7$, $b = 3$ and $s = 4$

$7.2^2 = 4.3^2 + 3.1^2 - 2(4.3)(3.1)\cos S$

$\cos S = \dfrac{4.3^2 + 3.1^2 - 7.2^2}{2(4.3)(3.1)} \approx -0.8905$

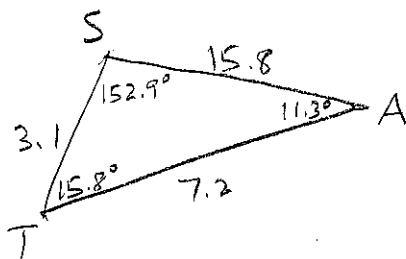$S \approx 152.9°$

$\dfrac{\sin T}{4.3} = \dfrac{\sin 152.9°}{7.2}$

$\sin T = \dfrac{4.3 \sin 152.9°}{7.2} \approx 0.2721$

$T \approx 15.8°$

$A \approx 180° - (152.9° + 15.8°)$
$= 11.3°$

$6.2 \sin 41° \approx 4.1$

$4.1 < 4.7 < 6.2$

$2 \triangle's$

$\dfrac{\sin R}{6.2} = \dfrac{\sin 41°}{4.7}$
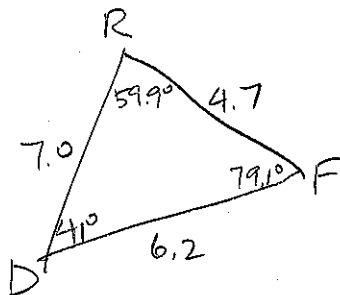
$\sin R = \dfrac{6.2 \sin 41°}{4.7} \approx 0.8654$

$R \approx 59.9°$ or $180° - 59.9° = 120.1°$

$\downarrow$ $\downarrow$

$F \approx 180° - (41° + 59.9°)$ $F \approx 180° - (41° + 120.1°)$
$= 79.1°$ $= 18.9°$

$\dfrac{f}{\sin 79.1°} = \dfrac{4.7}{\sin 41°}$ $\dfrac{f}{\sin 18.9°} = \dfrac{4.7}{\sin 41°}$

$f = \dfrac{4.7 \sin 79.1°}{\sin 41°} \approx 7.0$ $f = \dfrac{4.7 \sin 18.9°}{\sin 41°}$
≈ 2.3

$3 + 4 \ngtr 7$

DNE
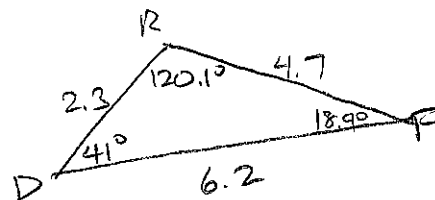
Suppose that $k = 12$ and $T = 26°$.

[a] Find all values of t for which there is exactly one possible triangle. **Do NOT solve the triangle.**

$$t = 12 \sin 26° \approx 5.3$$

$$\text{or } t \geq 12$$

[b] Find all values of t for which there are two possible triangles. **Do NOT solve the triangles.**

$$5.3 < t < 12$$

[c] Find all values of t for which there are no possible triangles.

$$t < 5.3$$

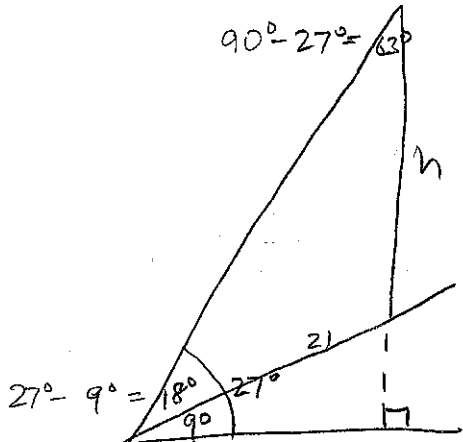Find the areas of the following triangles.   **SCORE: _____ / 4 PTS**

[a] ΔUVW if $m = 7.6$, $s = 9.1$, $M = 56.6°$ and $H = 34.8°$

$$\tfrac{1}{2}(7.6)(9.1)\sin 34.8° \approx 19.7$$

[b] ΔRPG if $r = 9$, $p = 4$ and $g = 7$

$$S = \tfrac{1}{2}(9+4+7) = 10 \qquad \sqrt{10(10-9)(10-4)(10-7)} = \sqrt{180} = 6\sqrt{5}$$

$$\approx 13.4$$

A flagpole at a right angle to the horizontal is located on a slope that makes an angle of $9°$ with the horizontal.   **SCORE: _____ / 8 PTS**
The flagpole's shadow is 21 feet long and points directly down the slope when the angle of elevation from the tip of the shadow to the sun
is $27°$. Find the height of the flagpole.

$$\frac{h}{\sin 18°} = \frac{21}{\sin 63°}$$

$$h = \frac{21 \sin 18°}{\sin 63°} \approx 7.3 \text{ ft}$$